of a reanalysis for small changes in the design parameters. The sensitivity derivatives of the flutter response to modal parameters are useful for identifying the modes that are important aeroelastically.

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Thermally Induced Vibration of a Symmetric Cross-Ply Plate with Hygrothermal Effects

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Introduction

THE response of filamentary composite plates to thermally induced vibrations when exposed to a hygrothermal environment is of major concern. The effects of heating on the dynamic response of plates were investigated as early as 1953 when Tsien[1] derived the general equations of motion for a heated, homogeneous, isotropic plate. Years later, Kao and Pao[2] developed the governing equations for a simply supported heterogeneous, anisotropic plate with rapid heating on one side. In the early 1970s, Whitney and Ashton[3] added expansional strains to the laminated plate equations. Over the next two decades, the effects of moisture on plate response have been studied using various models and techniques.[4–6] The present work studies the vibration of symmetric cross-ply plates under unsteady temperature effects and an unsteady moisture environment for the simply supported case.

Governing Equations

From classical laminated plate theory the transverse motion of a thin, symmetric, cross-ply plate exposed to elevated temperature and moisture conditions is

$$D_{11}w_{,xxxx} + 2(D_{12}+2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} + \rho h w_{,tt} = -M^T_{x,xx} - M^m_{x,xx} - M^T_{y,yy} - M^m_{y,yy} \tag{1}$$

in which ρ denotes the mass density, h is the thickness, D_{ij} are the laminate bending stiffnesses, and M^T_x, M^T_y, $M^m_{,x}$, and M^m_y represent the thermal and hygrothermal moments and are defined by

$$\begin{gathered}\left(M^T_x, M^T_y\right) = \sum_{k=1}^{R}\left[\left(\gamma_1^\alpha\right)_k, \left(\gamma_2^\alpha\right)_k\right]\int_{h_{k-1}}^{h_k}\Delta T z\,\mathrm{d}z\\ \left(\gamma_1^\alpha\right)_k = (\bar{Q}_{i1}\alpha_x + \bar{Q}_{i2}\alpha_y)_k\\ \left(M^m_x, M^m_y\right) = \sum_{k=1}^{R}\left[\left(\gamma_1^\beta\right)_k, \left(\gamma_2^\beta\right)_k\right]\int_{h_{k-1}}^{h_k}\Delta m z\,\mathrm{d}z\\ \left(\gamma_i^\beta\right)_k = (\bar{Q}_{i1}\beta_x + \bar{Q}_{i2}\beta_y)_k\end{gathered} \tag{2}$$

The thermal excitation considered is a suddenly applied heat flux[7]

$$\begin{gathered}\Delta T = \frac{hq_0}{k_z}\left[\frac{\beta^* t}{\pi^2} + \frac{1}{2}\left(\frac{z}{h}+\frac{1}{2}\right)^2 - \frac{1}{6} - \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{(-1)^j}{j^2}e^{-j^2\beta t}\cos j\pi\left(\frac{z}{h}+\frac{1}{2}\right)\right]\\ \beta* = \frac{\pi^2\alpha}{h^2}\end{gathered} \tag{3}$$

The hygrothermal excitation used is characterized by a sudden elevated moisture level on the upper plate surface

$$\begin{gathered}\Delta m = \delta_u + \sum_{r=0}^{\infty}B_r e^{-\lambda_r^2\beta t}\cos\lambda_r\left(z+\frac{h}{2}\right)\\ B_r = \frac{2}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}}\delta(z)\cos\lambda_r\left(z+\frac{h}{2}\right)\mathrm{d}z - \frac{4\delta_u(-1)^r}{(2r+1)\pi}\\ \delta_u = m_u - m_0, \qquad \delta(z) = m(z) - m_0\end{gathered} \tag{4}$$

Solution

An exact solution will be derived for a plate with all sides simply supported by assuming a deflection of the form

$$\begin{aligned}w &= w_1(x,y,t) + w_2(x,y,t)\\ &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(K_{mn}+q_{mn})\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\end{aligned} \tag{5}$$

Here w_1 (or K_{mn}) is the solution of Eq. (1) neglecting the inertia term and w_2 (or q_{mn}) is the solution of

$$D_{11}w_{2,xxxx} + 2(D_{12}+2D_{66})w_{2,xxyy} + D_{22}w_{2,yyyy} + \rho h w_{2,tt} = -\rho h w_{1,tt} \tag{6}$$

Letting $a_m = m\Pi/a$ and $b_n = n\Pi/b$ the thermal and hygrothermal moments are expressed as

$$\begin{aligned}&\left(M^T_x, M^T_y, M^m_x, M^m_y\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(B_{mn}, C_{mn}, D_{mn}, E_{mn})\sin a_m x\sin b_n y\\ &(B_{mn}, C_{mn}) = \frac{4(\cos m\pi - 1)(\cos n\pi - 1)}{mn\pi^2}\times\sum_{k=1}^{R}\left[\left(\gamma_1^\alpha\right)_k, \left(\gamma_2^\alpha\right)_k\right]\int_{h_{k-1}}^{h_k}\Delta T z\,\mathrm{d}z\\ &(D_{mn}, E_{mn}) = \frac{4(\cos m\pi - 1)(\cos n\pi - 1)}{mn\pi^2}\times\sum_{k=1}^{R}\left[\left(\gamma_1^\beta\right)_k, \left(\gamma_2^\beta\right)_k\right]\int_{h_{k-1}}^{h_k}\Delta m z\,\mathrm{d}zv\end{aligned} \tag{7}$$

The total response is then

$$K_{mn} = \left(\frac{1}{G_{mn}}\right)\left(B_{mn}a_m^2 + C_{mn}b_n^2 D_{mn}a_m^2 E_{mn}b_n^2\right)$$

$$G_{mn} = \left[D_{11}a_m^4 + 2(D_{12}+2D_{66})a_m^2 b_n^2 + D_{22}b_n^4\right]$$

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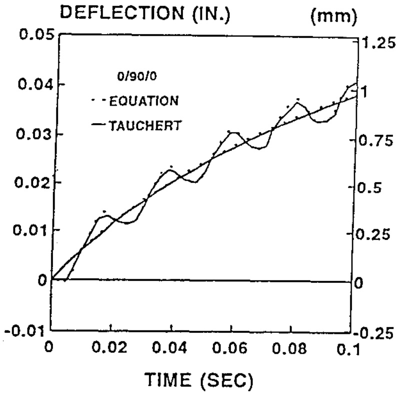
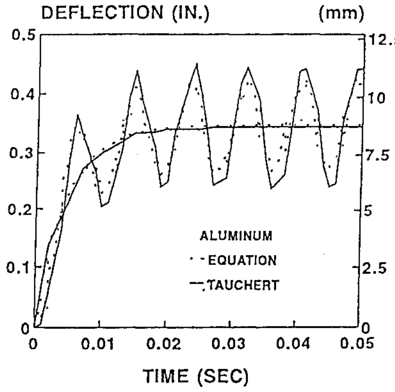
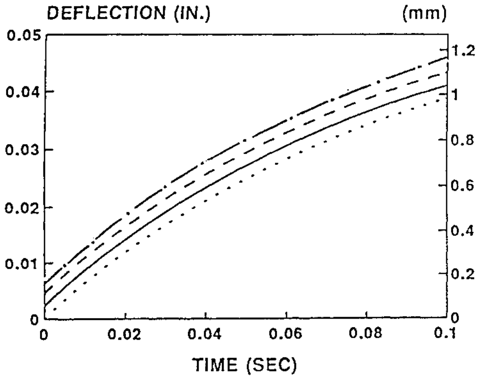


a)

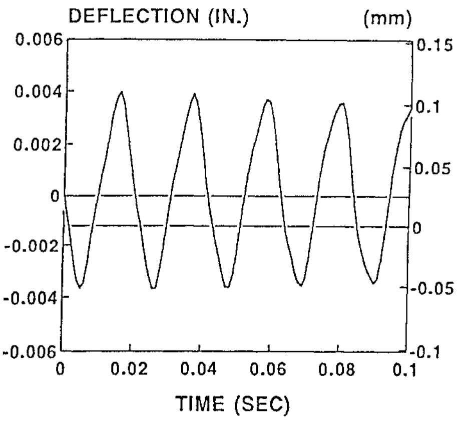


b)

Fig. 1 Comparison to previously published results.



a) Static



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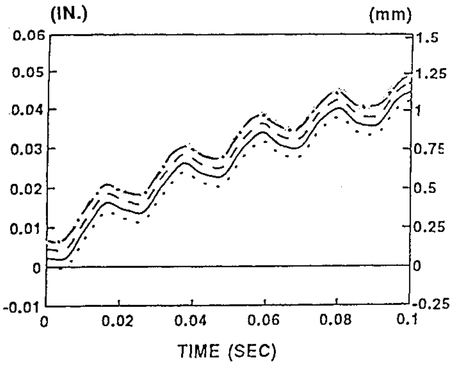

c) Total

Fig. 2 Plate response and moisture effects: 0/90/0, - - - -, 0.0%; ——, 0.2%; – – –, 0.4%; – · – · –, 0.6%.

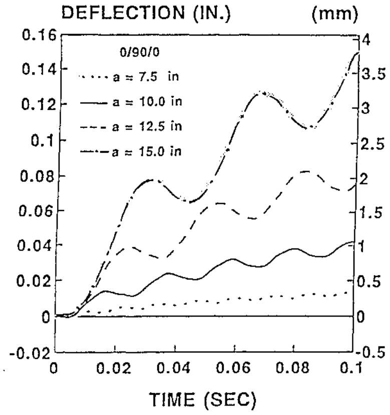

Fig. 3 Effect of aspect ratio on plate response.

$$
\begin{aligned}
q_{mn} = {} & \left(\sum_{j=1}^{\infty} \left[\frac{e^{-j^2\beta^* t} - \cos\omega_{mn} t}{j^4\beta^{*2} + \omega_{mn}^2} \right.\right. \\
& \left. - \frac{\omega_{mn}\sin\omega_{mn} t}{j^2\beta^*\left(j^4\beta^{*2} + \omega_{mn}^2\right)} \right] [S_{j,1}(z) + S_{j,2}(z)] \\
& + \sum_{r=0}^{\infty} \left\{ \left[\frac{\lambda_r^2\beta^2}{w_{mn}\left(\lambda_r^4\beta^2 + w_{mn}^2\right)} - \frac{1}{\lambda_r^2\beta\omega_{mn}} \right] \sin\omega_{mn} t \right. \\
& + \left[\frac{4\delta_u(-1)^r}{\lambda_r^4\beta^2 B_r(2r+1)\pi} + \frac{1}{\lambda_r^4\beta^2} - \frac{1}{\lambda_r^4\beta^2 + \omega_{mn}^2} \right] \cos\omega_{mn} t \\
& \left.\left. + \frac{e^{\lambda_r^2\beta t}}{\lambda_r^4\beta^2 + \omega_{mn}^2} \right\} [S_{r,3}(z) + S_{r,4}(z)] \right)
\end{aligned}
\tag{8}
$$

$$
[S_{j,1}(z), S_{j,2}(z), S_{r,3}(z), S_{r,4}(z)] = \frac{-1}{G_{mn}} \frac{\partial^2}{\partial t^2} (B_{mn}, C_{mn}, D_{mn}, E_{mn}) \bigg|_{t=0}
$$

Results and Discussion

Numerical results are obtained using AS/3501 graphite epoxy and aluminum. Material properties are given in Ref. 10. The temperature change is restricted to 100°F (55.6°C) as the material properties can no longer be considered temperature independent beyond this point. The plate considered consists of three layers and has a total thickness of 0.025 in. (0.635 mm).

Comparison with previously published work (Tauchert[8,9]) without the moisture effect is shown in Fig. 1. The present solution is seen to be in good agreement for both graphite epoxy and aluminum plates. Figure 2 shows the typical plate response. When a temperature gradient exists through the thickness, a quasistatic deflection is present, the magnitude of which is increased when a moisture gradient exists (Fig. 2a). As the moisture diffusion is extremely slow, the deflection due to the moisture is constant over long periods of time, and hence the moisture does not affect the dynamic motion (Fig. 2b). The total response of the plate represented by Eq. (5) is shown in Fig. 2c. Varying the aspect ratio causes an increase in the magnitude of the plate center displacements and oscillations and a decrease in frequency (Fig. 3).

Concluding Remarks

From the results obtained for the particular plates considered, moisture will only affect the quasistatic response due to its very slow diffusion into the material. The dynamic response is a function of temperature only.

As the aspect ratio is varied from one, the center deflection increases dramatically, and the oscillation increases in magnitude while decreasing in frequency.

The results obtained showed good agreement with published data for both the graphite epoxy and aluminum plates.

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Analytical Approach to Free Vibration of Sandwich Plates

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Nomenclature

D = bending stiffness factor of the plate
E_f = Young's modulus of face sheets
G_c = shear modulus of the core
T_i = surface forces
u_i = displacements
X_i = volume forces
ε_{ij} = strains
λ^{*2} = nondimensional eigenvalue, $\omega a^2(\rho/D)^{1/2}$
ν = Poisson's ratio of the face material
ρ = mass per unit area
σ_{ij} = stresses
ω = circular frequency of vibration

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I. Introduction

SANDWICH plates usually consist of three layers of which two outer thin sheets are of high-strength material and a core is of low strength. In several applications, the thin skin sheets are made of aluminum, titanium, heat-resistant steel, or other materials such as plywood, hardboard, or reinforced plastics. The intermediate core serves to keep the facial layers apart and may also act as a thermal barrier.

Because of their high strength to weight ratios and good thermal and acoustical insulation properties, sandwich plate construction has become increasingly useful in various areas of structural design, especially in the aerospace industry, e.g., in fins, wings, and fuselage and pressure bulkheads.

In one of the earliest attempts at analysis, Reissner[1] developed the basic differential equations for the buckling of plates by using a simplified model consisting of two facings as membranes and a core resisting shear and normal stresses. Falgout[2] obtained the differential equations for free vibration of sandwich plates with isotropic faces and core by superposing the bending deflections and the deflections due to transverse shears. Several other authors used the finite element method to study the dynamic behavior of sandwich plates. Ahmed[3] employed the finite element technique to solve the vibration problem of a doubly curved honeycomb sandwich plate. Ng and Das[4] studied the vibration and buckling of clamped skew sandwich plates by the Galerkin method.

The purpose of this paper is to present an analytical approach, based on the reciprocal theorem,[5,6] for the free vibration of sandwich plates. It will be seen that the method works quite well, and excellent agreement is found between the results reported here and those published in earlier references.

II. Governing Equation

The theory presented here has been developed within the framework of linear theory and small displacements. Furthermore, we consider the face and core materials to be homogeneous and isotropic, the two face sheets to be of equal thickness t and the core layer to have a constant thickness c.

The differential equation for the free vibration of the plate in terms of the lateral displacement $w(x, y, t)$ can be written as[2]

$$\nabla^4 w(x, y, t) = \rho\left[\nabla^2 w(x, y, t)_{,tt}/(G_c C) w(x, y, t)_{,tt}/D\right] \quad (1)$$

where

$$\nabla^2 = (\cdot)_{xx} + (\cdot)_{,yy}, \qquad D = E_f t^3(1 + 3h^2/t^2)/[6(1 - v^2)]$$

$$h = c + t$$

If the plate is subjected to a time varying force $f(x, y, t)$, the equation governing the motion of the plate can be obtained by expressing the force and displacement as a product of two functions, one involving only spatial coordinates and the other involving the time t, as follows:

$$\nabla^4 W(x, y) + \lambda^4[(D/G^*)\nabla^2 W(x, y) - W(x, y)]$$
$$= F(x, y)/D - \nabla^2 F(x, y)/G^* \quad (2)$$

where $G^* = G_c c$ and frequency coefficient λ^2 can be written as $\lambda^2 = \omega\sqrt{(\rho/D)}$,

III. Reciprocal Theorem

The reciprocal theorem states that if a Hookean body is exposed to two different systems of volume and surface forces, then the actual work done by the forces of the first system along the displacements of the second system is equal to the work done by the forces of the second system along the displacements belonging to the first system.